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VIBROWAVE ACTION ON THE BED AND THE CRITICAL AREA OF WELLS WITH ALLOWANCE FOR THE SLIP EFFECT

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The model of the process is constructed and the factors influencing its mechanism are determined. It is shown that the influence of vibrowave action on the process of filtration is significantly improved in the presence of the slip effect. The depth of propagation of elastic waves in the bed as a function of their velocity is determined.

Introduction. One efficient method of raising the productivity of wells is vibrowave action on the bed and the critical area. For this purpose one installs a generator on the flow of the fluid produced; the generator creates elastic waves acting on the bed and the critical area and contributing to the development of microcracks in oil-bearing formations and influencing the rheological properties of the fluid [1, 2].

This problem has been the focus of the works of a number of authors. However, the mechanism of vibroaction on the bed and the critical area is complicated and little known [3–7]. There is no clear understanding of the factors determining its efficiency [7]. In this work, we construct a model of the process and determine the factors influencing its mechanism.

Formulation of the Problem. We consider a simplified model and different cases of pulsations of the well flow rate which are created by the generator (Fig. 1). It is assumed that the permeability of the formation is dependent on pressure. Then the pressure distribution in the formation is described by the piezoconductivity equation [4]

$$\frac{\partial P}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[\chi(P)\, r\,\frac{\partial P}{\partial r}\right]. \tag{1}$$

The initial and boundary conditions are as follows:

$$P_{\mathrm{g}}\big|_{t=0} = P_{\mathrm{b}}\,, \tag{2}$$

$$P\big|_{r=R_{\mathrm{ex}}} = P_{\mathrm{ex}}\,, \tag{3}$$

$$2\pi r h \frac{k}{\mu}\frac{\partial P}{\partial r}\bigg|_{r=r_{\mathrm{w}}} = Q(t)\,. \tag{4}$$

We apply the Kirchhoff transformation [5] to solution of the nonlinear boundary-value problem (1)

$$\theta = \frac{1}{\chi_0}\int_0^P \chi(P)\, dP\,. \tag{5}$$

From (1), with account for (4), we obtain

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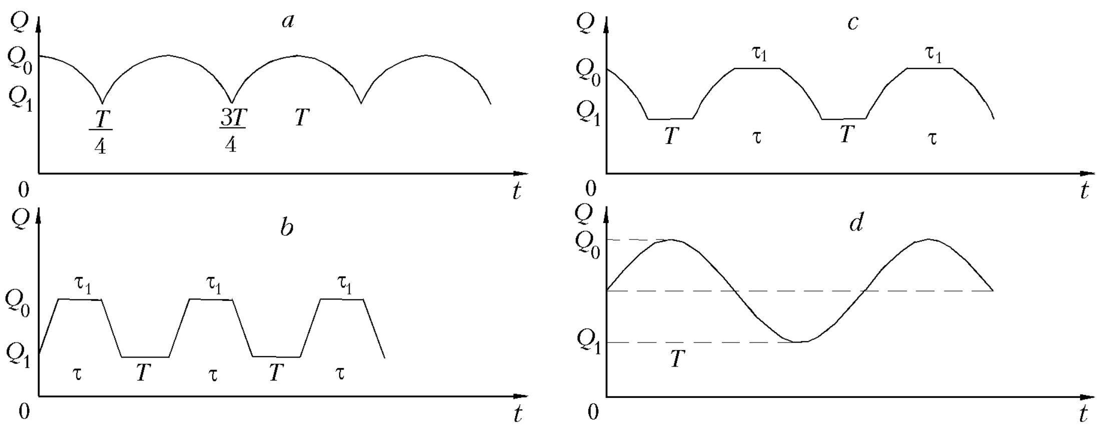

Fig. 1. Different forms of elastic waves.

$$\frac{\partial\theta}{\partial t} = \chi_0 \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial\theta}{\partial r}\right]. \tag{6}$$

Exact Solution. Based on experimental investigations, it has been shown in [2] that near (above) the saturation pressure, the formation of surface gas nuclei leads to a slip effect, because of which the bed permeability changes. The effective permeability in this case can be determined from the formula

$$k(P) = k_0\left[1 + \frac{4b(P)}{R}\right]. \tag{7}$$

Next, we consider the filtration process above and near the saturation pressure (without allowance for the slip effect and with allowance for it). Linearizing the results obtained in [2], for the piezoconductivity we obtain

$$\chi = \frac{k_0}{\mu\beta^*}\left[1 + \frac{4}{R}a(P - P_{\rm w})\right], \quad P_{\rm w} \le P \le P_{\rm m}; \quad \chi = \frac{k_0}{\mu\beta^*}\left[1 + \frac{1}{R}a(P_{\rm s} - P)\right], \quad P_{\rm m} \le P \le P_{\rm s}. \tag{8}$$

From (5), with account for (8), we have

$$\theta = P + \frac{2a}{R}\left[(P - P_{\rm w})^2 - P_{\rm w}^2\right], \quad P_{\rm w} \le P \le P_{\rm m}; \quad \theta = P + \frac{a}{2R}\left[P_{\rm s}^2 - (P_{\rm s} - P)^2\right], \quad P_{\rm m} \le P \le P_{\rm s}. \tag{9}$$

Boundary condition (4), with account for (7) and (9), will take the form

$$\left.\frac{k(P)}{\mu}2\pi hr\frac{\partial P}{\partial r}\right|_{r=r_{\rm w}} = \left.\frac{k_0}{\mu}2\pi hr\frac{\partial\theta}{\partial r}\right|_{r=r_{\rm w}}. \tag{10}$$

Equation (5), with account for initial (2) and boundary conditions (3) and (4), can be solved using the Duhamel integral [8]. With account for (10), we represent it as [9]

$$\Delta\theta(r;t) = Q(0)\,\Delta\theta_1(r;t) + \int_0^t \dot{Q}(\tau)\,\Delta\theta_1(r, t-\tau)\,d\tau, \tag{11}$$

where $\Delta\theta_1$ is the solution of Eq. (5) with initial (2) and boundary conditions (3) and (10) for $Q = Q(0) = \text{const}$. The solution of (5), with account for (2), (3), (10), and $Q = Q(0) = \text{const}$, has the form [9]

$$\Delta\theta_1(r;t)=\frac{Q(0)\,\mu}{2\pi b k_0}\left[\ln\frac{R_{ex}}{r}+\pi\sum_{\nu=1}^{\infty}\frac{1}{x_\nu}\frac{J_0\left(x_\nu\frac{R_{ex}}{r_w}\right)J_1(x_\nu)\,U_\nu\left(x_\nu\frac{r}{r_w}\right)}{J_1^2(x_\nu)-J_0^2\left(x_\nu\frac{R_{ex}}{r_w}\right)}\exp\left(-x_\nu^2\frac{\chi_0 t}{r_w^2}\right)\right], \tag{12}$$

where x_ν are the roots of the transcendental equation

$$J_0\left(x_\nu\frac{R_{ex}}{r_w}\right)Y_1(x_\nu)-J_1(x_\nu)\,Y_0\left(x_\nu\frac{R_{ex}}{r_w}\right)=0\,;\quad U_\nu\left(x_\nu\frac{r}{r_w}\right)=J_0\left(x_\nu\frac{r}{r_w}\right)Y_0\left(x_\nu\frac{R_{ex}}{r_w}\right)-J_0\left(x_\nu\frac{R_{ex}}{r_w}\right)Y_0\left(x_\nu\frac{r}{r_w}\right). \tag{13}$$

Integration of (11), with account for (12), at different $Q(t)$ values enables us to find the pressure distribution $P(r, t)$ over the bed. Using the elastic-wave generator we can create, on the bottom, different forms of elastic waves (Fig. 1a–d)) whose equations will respectively be represented as

$$Q=Q_1+(Q_0-Q_1)\left|\cos\frac{2\pi}{T}t\right|, \tag{14}$$

$$\begin{aligned}Q=\Bigg\{&\frac{2(Q_0-Q_1)}{\tau-\tau_1}t\left[\eta(t)-\eta\left(t-\frac{\tau-\tau_1}{2}\right)\right]+(Q_0-Q_1)\left[\eta\left(t-\frac{\tau-\tau_1}{2}\right)-\eta\left(t-\frac{\tau+\tau_1}{2}\right)\right]\\&+\left[Q_0-Q_1-\frac{2(Q_0-Q_1)}{\tau-\tau_1}(\tau-t)\right]\left[\eta\left(t-\frac{\tau+\tau_1}{2}\right)-\eta(t-\tau)\right]\Bigg\}\\&\times\Big\{\eta\left[t-(\nu-1)(T+\tau)\right]-\eta\left[t-\nu\tau+(\nu-1)T\right]\Big\}+Q_1,\end{aligned} \tag{15}$$

$$\begin{aligned}Q=\Bigg\{&(Q_0-Q_1)\sin\left(\frac{\pi}{\tau-\tau_1}t\right)\left[\eta(t)-\eta\left(t-\frac{\tau-\tau_1}{2}\right)\right]+(Q_0-Q_1)\left[\eta\left(t-\frac{\tau-\tau_1}{2}\right)-\eta\left(t-\frac{\tau+\tau_1}{2}\right)\right]\\&+(Q_0-Q_1)\cos\left[\frac{\pi}{\tau-\tau_1}\left(t-\frac{\tau+\tau_1}{2}\right)\right]\left[\eta\left(t-\frac{\tau+\tau_1}{2}\right)-\eta(t-\tau)\right]\Bigg\}\\&\times\Big\{\eta\left[t-(\nu-1)(T+\tau)\right]-\eta\left[t-\nu\tau+(\nu-1)T\right]\Big\}+Q_1,\end{aligned} \tag{16}$$

$$Q=Q_1+\frac{Q_0-Q_1}{2}\cos\left(\frac{2\pi}{T}t\right). \tag{17}$$

Then, from (11) with account for (12) and the action function (17) which occurs in practice in most cases, we obtain

$$\Delta\theta=\frac{(Q_0+Q_1)\,\mu}{4\pi h k_0}\left[\ln\frac{R_{ex}}{r}+\pi\sum_{\nu=1}^{\infty}\frac{1}{x_\nu}\frac{J_0\left(x_\nu\frac{R_{ex}}{r_w}\right)J_1(x_\nu)\,U_\nu\left(x_\nu\frac{r}{r_w}\right)}{J_1^2(x_\nu)-J_0^2\left(x_\nu\frac{R_{ex}}{r_w}\right)}\exp\left(-x_\nu^2\frac{\chi_0 t}{r_w^2}\right)\right]$$

$$+\frac{(Q_0-Q_1)\,\mu}{4\pi h k_0}\ln\frac{R_{ex}}{r}\left(\cos\frac{2\pi}{T}t-1\right)-\frac{\pi}{T}(Q_0-Q_1)\frac{\mu}{2hk_0}\sum_{\nu=1}^{\infty}\frac{1}{x_\nu}\frac{J_0\left(x_\nu\frac{R_{ex}}{r_w}\right)J_1(x_\nu)\,U_\nu\left(x_\nu\frac{r}{r_w}\right)}{J_1^2(x_\nu)-J_0^2\left(x_\nu\frac{R_{ex}}{r_w}\right)}$$

$$\times\left(\frac{\frac{2\pi}{T}\exp\left(-x_\nu^2\frac{\chi_0 t}{r_w^2}\right)}{\frac{x_\nu^4\chi_0^2}{r_w^4}+\frac{4\pi^2}{T^2}}+\frac{\frac{x_\nu^2\chi_0}{r_w^2}\sin\frac{2\pi}{T}t-\frac{2\pi}{T}\cos\frac{2\pi}{T}t}{\frac{x_\nu^4\chi_0^2}{r_w^4}+\frac{4\pi^2}{T^2}}\right). \qquad (18)$$

On the condition that $r_w \le 0.02R_{ex}$ which is virtually always observed in the case in question, formula (18) is simplified and takes the form [10]

$$\Delta\theta=\frac{Q_0-Q_1}{2}\frac{\mu}{2\pi h k_0}\ln\left(\frac{R_{ex}}{r}\right)\left(\cos\frac{2\pi}{T}t-1\right)+\frac{\pi}{T}(Q_0-Q_1)\frac{\mu}{hk_0}\left(\frac{r_w}{R_{ex}}\right)^2$$

$$\times\sum_{\nu=1}^{\infty}\frac{J_0\left(x_\nu\frac{r}{r_w}\right)}{x_\nu^2J_1^2\left(x_\nu\frac{R_{ex}}{r_w}\right)}\left(\frac{\frac{2\pi}{T}\exp\left(-x_\nu^2\frac{\chi_0 t}{r_w^2}\right)}{\frac{x_\nu^4\chi_0^2}{r_w^4}+\frac{4\pi^2}{T^2}}+\frac{\frac{x_\nu^2\chi_0}{r_w^2}\sin\frac{2\pi}{T}t-\frac{2\pi}{T}\cos\frac{2\pi}{T}t}{\frac{x_\nu^4\chi_0^2}{r_w^4}+\frac{4\pi^2}{T^2}}\right)$$

$$+\frac{(Q_0+Q_1)\,\mu}{4\pi h k_0}\left[\ln\frac{R_{ex}}{r}-2\left(\frac{r_w}{R_{ex}}\right)^2\sum_{\nu=1}^{\infty}\frac{J_0(x_\nu)}{x_\nu^2J_1^2\left(x_\nu\frac{R_{ex}}{r_w}\right)}\exp\left(-x_\nu^8\frac{\chi_0 t}{r_w^2}\right)\right], \qquad (19)$$

where x_ν are the roots of the transcendental equation

$$J_0\left(x_\nu\frac{R_{ex}}{r_w}\right)=0\,,\quad \Delta\theta=\theta_{ex}-\theta\,. \qquad (20)$$

The value of θ_{ex} is determined from (9) and has the form

$$\theta_{ex}=P_{ex}+\frac{2a}{R}\left[(P_{ex}-P_w)^2-P_w^2\right],\quad P_w\le P\le P_m;\quad \theta_{ex}=P_{ex}+\frac{a}{2R}\left[P_s^2-(P_s-P_{ex})^2\right],\quad P_m\le P\le P_s\,.$$

Then for the pressure field in the bed with allowance for the slip effect we obtain, from (9),

$$P=\frac{-\left(1-\frac{4a}{R}P_w\right)+\sqrt{\left(1-\frac{4a}{R}P_w\right)^2+\frac{8a}{R}\theta}}{\frac{4a}{R}},\quad P_w\le P\le P_m\,; \qquad (21)$$

$$P = \frac{1 + \frac{a}{R} P_s - \sqrt{\left(1 + \frac{a}{R} P_s\right)^2 - \frac{2a}{R}\theta}}{\frac{a}{R}}, \quad P_m \le P \le P_s, \tag{22}$$

where $\theta = \theta_{ex} - \Delta\theta$, $\Delta\theta$ is determined from formula (18) or (19).

When the slip of the fluid is absent, from (1), with account for (2)–(4) and (19), for the pressure field in the bed we have

$$P = P_{ex} - \frac{(Q_0 - Q_1)\,\mu}{4\pi h k_0} \ln\left(\frac{R_{ex}}{r}\right)\left(\cos\frac{2\pi}{T}t - 1\right) - \frac{\pi}{T}\frac{(Q_0 - Q_1)\,\mu}{h k_0}\left(\frac{r_w}{R_{ex}}\right)^2$$

$$\times \sum_{\nu=1}^{\infty} \frac{J_0\left(x_\nu \frac{r}{r_w}\right)}{x_\nu^2 J_1^2\left(x_\nu \frac{R_{ex}}{r_w}\right)} \left(\frac{\frac{2\pi}{T}\exp\left(-x_\nu^2 \frac{\chi_0 t}{r_w^2}\right)}{\frac{x_\nu^4 \chi_0^2}{r_w^4} + \frac{4\pi^2}{T^2}} + \frac{\frac{x_\nu^2 \chi_0}{r_w^2}\sin\frac{2\pi}{T}t - \frac{2\pi}{T}\cos\frac{2\pi}{T}t}{\frac{x_\nu^2 \chi_0^2}{r_w^4} + \frac{4\pi^2}{T^2}}\right)$$

$$- \frac{\frac{Q_0 + Q_1}{2}\mu}{2\pi h k_0}\left[\ln\frac{R_{ex}}{r} - 2\left(\frac{r_w}{R_{ex}}\right)^2 \sum_{\nu=1}^{\infty} \frac{J_0(x_\nu)}{x_\nu^2 J_1^2\left(x_\nu \frac{R_{ex}}{r_w}\right)} \exp\left(-x_\nu^2 \frac{\chi_0 t}{r_w^2}\right)\right]. \tag{23}$$

Approximate Solution. With the aim of simplifying the resulting formulas (18), (19), and (23) we find approximately the solution of Eq. (5) with an accuracy sufficient for engineering calculations. Averaging $\partial\theta/\partial t$ over r, we will have

$$\varphi(t) = \frac{2}{R_{ex}^2 - r_w^2} \int_0^{R_{ex}} \frac{\partial\theta}{\partial t} r dt. \tag{24}$$

Substituting (24) into (5) and integrating the resulting expression with account for (2)–(4) and (10), we obtain

$$\theta = \theta_{ex} + \frac{\varphi(t)}{4\chi_0}(r^2 - R_{ex}^2) - \frac{\varphi(t)}{2\chi_0} r_w^2 \ln\left(\frac{r}{R_{ex}}\right) + \frac{\mu Q(t)}{2\pi h k_0} \ln\frac{r}{R_{ex}}. \tag{25}$$

From (24), with account for (25), we have, disregarding r_w^2,

$$\dot{\varphi} + \frac{8\chi_0}{R_{ex}^2}\varphi = -\frac{2\dot{Q}(t)}{\pi\beta^* h R_{ex}^2}. \tag{26}$$

Integrating (26), we can find the pressure distribution over the formation as a function of r and t for different cases of variation in $Q(t)$. As an example, we also consider the function of action on the critical area and the bed (17). Then, from (26), with account for (17) and (25) and the initial condition

$$\theta\big|_{t=0} = \theta_{ex} + \frac{\mu(Q_1 + Q_0)}{4\pi h k_0} \ln\frac{r}{R_{ex}}$$

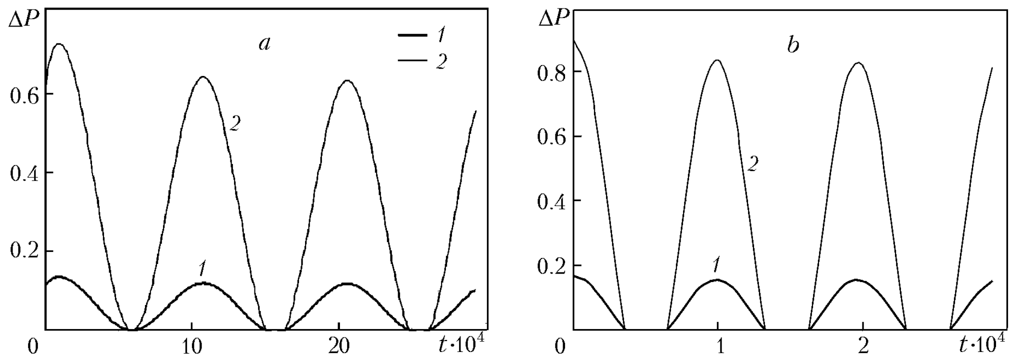

Fig. 2. Dynamics of the pressure difference ΔP_g for P_{ex} = 200 m: a) according to the exact solution; b) according to the approximate solution; 1) with allowance for the slip effect; 2) without allowance for it. t, sec; ΔP, MPa.

we obtain

$$\theta = \frac{r^2 - R_{ex}^2}{4\chi_0} \frac{Q_0 - Q_1}{\pi\beta^* h R_{ex}^2} \left\{ -\cos\omega t + \frac{1}{R_{ex}^2} \frac{8\chi_0}{\omega^2 + \left(\frac{8\chi_0}{R_{ex}^2}\right)^2} \left(\frac{8\chi_0}{R_{ex}^2} \cos\omega t + \omega \sin\omega t \right) \right.$$

$$\left. + \left[1 - \frac{\left(\frac{8\chi_0}{R_{ex}^2}\right)^2}{\omega^2 + \left(\frac{8\chi_0}{R_{ex}^2}\right)^2} \right] \exp\left(-\frac{8\chi_0 t}{R_{ex}^2} \right) \right\} + \frac{\mu}{2\pi h k_0} \ln\frac{r}{R_{ex}} \left(Q_1 + \frac{Q_0 - Q_1}{2} \cos\omega t \right) + \theta_{ex}, \quad (27)$$

where $\omega = 2\pi/T$.

Next, the pressure distribution over the bed with allowance for the fluid's slip is determined from formulas (21) and (22) for the function θ determined by Eq. (27). When slip is absent ($P > P_s$), the pressure distribution over the bed is also determined from (27) and has the form

$$P = P_{ex} + \frac{r^2 - R_{ex}^2}{4\chi_0} \frac{Q_0 - Q_1}{\pi\beta^* h R_{ex}^2} \left\{ -\cos\omega t + \frac{1}{R_{ex}^2} \frac{8\chi_0}{\omega^2 + \left(\frac{8\chi_0}{R_{ex}^2}\right)^2} \left(\frac{8\chi_0}{R_{ex}^2} \cos\omega t + \omega \sin\omega t \right) \right.$$

$$\left. + \left[1 - \frac{\left(\frac{8\chi_0}{R_{ex}^2}\right)^2}{\omega^2 + \left(\frac{8\chi_0}{R_{ex}^2}\right)^2} \right] \exp\left(-\frac{8\chi_0 t}{R_{ex}^2} \right) \right\} + \frac{\mu}{2\pi h k_0} \ln\frac{r}{R_{ex}} \left(Q_1 + \frac{Q_0 - Q_1}{2} \cos\omega t \right). \quad (28)$$

The bottom-hole pressure can be determined from (21) and (22) with account for (27) and from (28) for $r = r_w$.

Discussion of Results. As is seen from (19), (23), and (28), the waves created by the generator are attenuated as they bite deeper into the bed. The attenuation of the waves is strongly dependent on the frequency ω. As the fre-

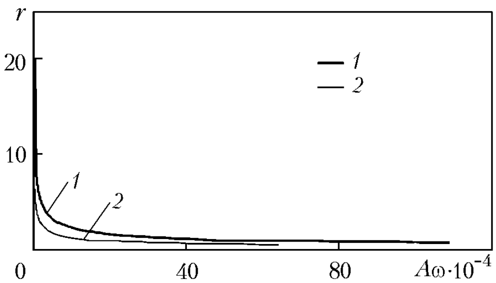

Fig. 3. Depth of penetration of elastic waves vs. vibrational velocity ($A\omega$) $\cdot 10^{-4}$ m^3/sec: 1) exact solution; 2) approximate solution. $A\omega$, m^3/sec^2; r, m.

quency of the waves decreases, the depth of their penetration into the bed increases, which is of great practical importance. The propagation of elastic waves with allowance for the slip is nearly of the same character as that without allowance for it. Furthermore, to increase the depth of penetration of the waves we must select their frequency to be a multiple of $8\chi_0/R_{ex}^2$.

Figures 2 and 3 give results of numerical calculations, according to the exact and approximate solutions, of the depth of penetration of elastic waves for different $A\omega$ and of the dynamics of the pressure difference for the following values of the system's parameters: $P_{ex} = 10^7$ Pa, $\mu = 1$ mPa·sec, $h = 10$ m, $r_w = 0.15$ m, $\chi_0 = 0.17$ m^2/sec, $k_0 = 0.5 \cdot 10^{-12}$ m^2, $a = 10^{-12}$ m/Pa, $R = 4.47 \cdot 10^{-6}$ m, $\beta^* = 2.94 \cdot 10^{-9}$ 1/Pa, $R_{ex} = 200$ m, $P_3 = 8.5 \cdot 10^6$ Pa, and $P_w = 5 \cdot 10^6$ Pa.

From Fig. 2, it is clear that, all other things being equal, the pressure difference in the process of vibrowave action with the slip effect is always lower than that without it, which is of great practical importance. Indeed, this enables one to maintain a high drainage for relatively low pressure differences [2].

As the frequency of elastic waves decreases, the depth of their penetration into the bed increases (see Fig. 3). As the quantity $A\omega$ grows to 10^{-3} m^3/sec^2, the depth of penetration of elastic waves sharply decreases, following which it becomes stabilized and is weakly dependent on $A\omega$. The largest effect from the action is obtained in the interval $0 \le A\omega \le 10^{-3}$ m^3/sec^2.

A comparison of the results of calculations carried out according to the exact and approximate solutions has shown that the approximate solution yields values of the pressure differences (Fig. 2) and the depth of penetration of elastic waves that are overstated by 20%. This enables one to use the approximate solution for practical engineering calculations.

NOTATION

A, vibration amplitude, $A = \frac{Q_0 + Q_1}{2}$; a, constant coefficient; b, slip coefficient; $k(p)$, effective permeability, m^2; h, bed thickness, m; n, constant quantity; P, pressure at any point of the bed, MPa; P_g, pressure on the bed gallery, MPa; P_m, pressure at which the permeability coefficient takes its maximum value, MPa; P_b, bottom-hole pressure, MPa; P_{ex}, pressure at the external boundary of the bed, MPa; P_w, saturation pressure, MPa; P_s, pressure at which the action of the slip effect ceases, MPa; $Q(t)$, flow rate, m^3/sec; Q_0, upper limiting value of the well flow rate, m^3/sec; $\dot{Q}(t)$, derivative of $Q(t)$; Q_1, lower limiting value of the well flow rate, m^3/sec; R, average radius of the pore channel, m; R_{ex}, radius of the external boundary of the bed, m; r, coordinate, m; r_w, radius of the well, m; T, period of vibration, sec; t, time, sec; J_0 and J_1 and Y_0 and Y_1, Bessel functions of the first and second kind and of zero and first order; β^*, compressibility coefficient, 1/Pa; χ, piezoconductivity coefficient, m^2/sec; χ_0, initial value of the piezoconductivity coefficient, m^2/sec; η, Heaviside function; $\varphi(t)$, unknown function; φ, derivative of $\varphi(t)$; μ, coefficient of viscosity of the fluid, mPa·sec; $\nu = 1, 2, 3, ..., n$, natural numbers; θ, Kirchhoff function; τ and τ_1, times characterizing the rate of pulse action, sec; ω, frequency of vibrations of elastic waves, sec^{-1}. Subscripts: g, gallery; m, maximum value; b, bottom hole; ex, external boundary; w, well; st, stationary; s, cessation of the action of the slip effect; 0 and 1, upper and lower value.

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